

Démo

$$\text{ch}(2x) = \frac{e^{2x} + e^{-2x}}{2}$$

$$\text{ch}^2 x + \text{sh}^2 x = \left(\frac{e^x + e^{-x}}{2} \right)^2 + \left(\frac{e^x - e^{-x}}{2} \right)^2$$
$$= \frac{(e^x)^2 + 2e^x e^{-x} + (e^{-x})^2 + (e^x)^2 - 2e^x e^{-x} + (e^{-x})^2}{4}$$

$$= \frac{e^{2x} + \cancel{2} + e^{-2x} + e^{2x} - \cancel{2} + e^{-2x}}{4}$$

$$= \frac{2e^{2x} + 2e^{-2x}}{4}$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

$$\Rightarrow \boxed{\text{ch}(2x) = \text{ch}^2 x + \text{sh}^2 x}$$

Démo

$$\bullet \text{sh}(2x) = \frac{e^{2x} - e^{-2x}}{2}$$

$$\begin{aligned}
 2 \operatorname{ch} x \operatorname{sh} x &= 2 \left(\frac{e^x + e^{-x}}{2} \right) \left(\frac{e^x - e^{-x}}{2} \right) \\
 &= \frac{1}{2} \left((e^x)^2 - (e^{-x})^2 \right) \\
 &= \frac{1}{2} \left(e^{2x} - e^{-2x} \right)
 \end{aligned}$$

$$\Rightarrow \boxed{\operatorname{sh}(2x) = 2 \operatorname{ch} x \operatorname{sh} x}$$

$$\begin{aligned}
 \operatorname{th} x &= \frac{\operatorname{sh} x}{\operatorname{ch} x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \stackrel{(xe^x)}{=} \frac{e^{2x} - 1}{e^{2x} + 1} \\
 &= \frac{1 - e^{-2x}}{1 + e^{-2x}}
 \end{aligned}$$

$$\cdot D_f = \mathbb{R}$$

$$\cdot D_c \oplus \forall x \in D_f, -x \in D_f$$

$$\begin{aligned}
 f(-x) &= \frac{\operatorname{sh}(-x)}{\operatorname{ch}(-x)} = \frac{-\operatorname{sh}(x)}{\operatorname{ch}(x)} \\
 &= -f(x)
 \end{aligned}$$

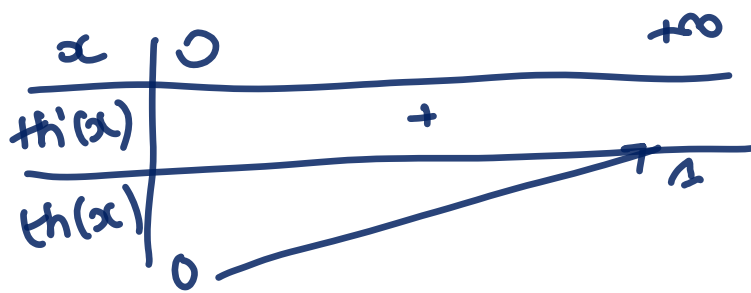
car
sh est
impaire,
ch est
paire

\Rightarrow th est impaire

th est dérivable $\hat{=}$ quotient

$$\operatorname{th}'(x) = \frac{\operatorname{ch} x \cdot \operatorname{ch} x - \operatorname{sh} x \cdot \operatorname{sh} x}{\operatorname{ch}^2 x} = \frac{\operatorname{ch}^2 x - \operatorname{sh}^2 x}{\operatorname{ch}^2 x}$$

$$\text{th}'(x) = 1 - \text{th}^2 x = \frac{1}{\text{ch}^2 x}$$



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