DS n°2 M 1.2 2022

Exercice 1 :

$f(x) = \ln(ax^2 + 5)$ $\quad a \in \mathbb{R}$

1)

Df : $ax^2 + 5 > 0$

$ax^2 > -5$

① **si $a > 0$** $\quad Df = \mathbb{R}$. $\quad x^2 > \frac{-5}{a}$ toujours vrai.

② **si $a < 0$** $\quad x^2 < -\frac{5}{a}$ et $Df = \left]-\sqrt{\frac{-5}{a}} ; \sqrt{\frac{-5}{a}}\right[$

2)

① **si $a > 0$** $\quad \lim\limits_{\pm\infty} ax^2 = +\infty$ et $\lim\limits_{\pm\infty} f(x) = +\infty$

② **si $a < 0$** $\quad \lim\limits_{x \to \pm\sqrt{\frac{-5}{a}}} ax^2 + 5 = 0^+$ et $\lim f(x) = -\infty$

3) **si $a < 0$:** asymptotes verticales

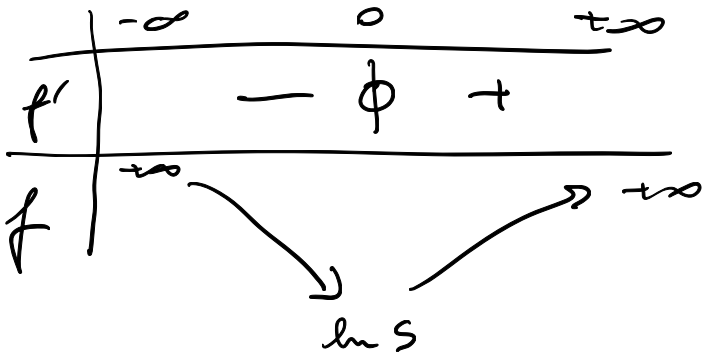
si $a > 0$: $\dfrac{f(x)}{x} = \dfrac{\ln(ax^2 + 5)}{x}$

Th de l'hospital : $\lim_{\pm\infty} \frac{f(x)}{x} = \lim_{\pm\infty} \frac{2ax}{ax^2+S} = 0$

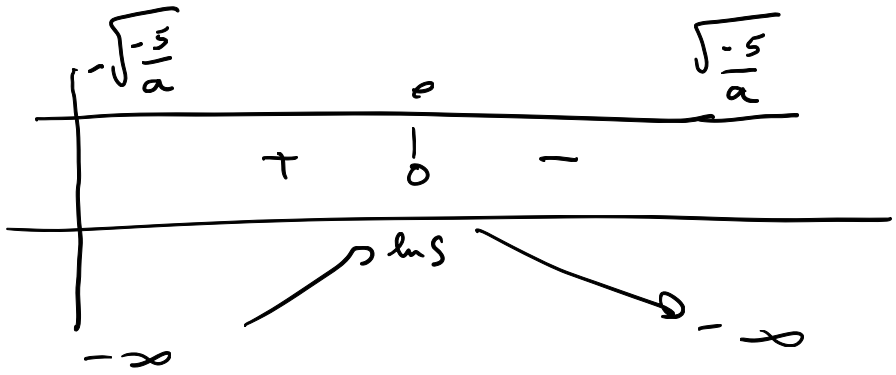
branche parabolique de direction Ox

4] $f'(x) = \frac{2ax}{ax^2+S}$

si $a>0$

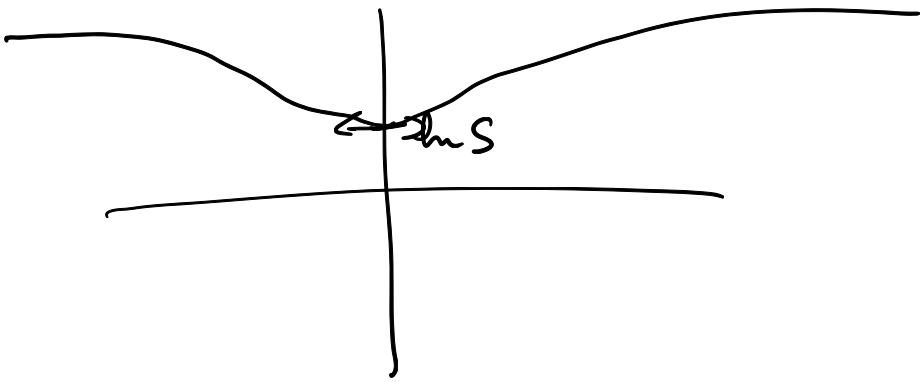


si $a<0$

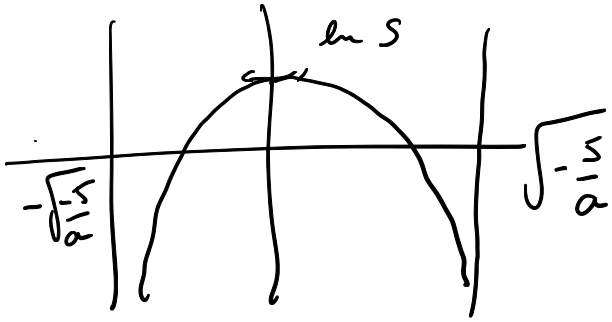


5]

si $a>0$



si $a<0$



Exercice 2 :

$$f(x) = \frac{2x^2 - 3x + 5}{5x - 4}$$

$$Df = \mathbb{R} \setminus \left\{ \frac{4}{5} \right\}$$

$\lim\limits_{+\infty} f(x) = \lim \frac{2}{5} x = +\infty$ $\qquad$ $\lim\limits_{x \to -\infty} f(x) = -\infty$.

$$\frac{f(x)}{x} = \frac{2x^2 - 3x + 5}{5x^2 - 4x}$$

$$\lim_{\pm\infty} \frac{f(x)}{x} = \frac{2}{5}$$

$$f(x) - \frac{2}{5}x = \frac{2x^2 - 3x + 5}{5x - 4} - \frac{2}{5} x$$

$$= \frac{1}{5x - 4} \left(2x^2 - 3x + 5 - 2x^2 + \frac{8}{5} x \right)$$

$$= \frac{-\frac{7}{5} x + 5}{5x - 4}$$

et $\lim\limits_{\pm\infty} f(x) - \frac{2}{5} x = -\frac{7}{25}$

$y = \frac{2}{5} x - \frac{7}{25}$ est une asymptote oblique en $\pm\infty$.

Exercice 3 :

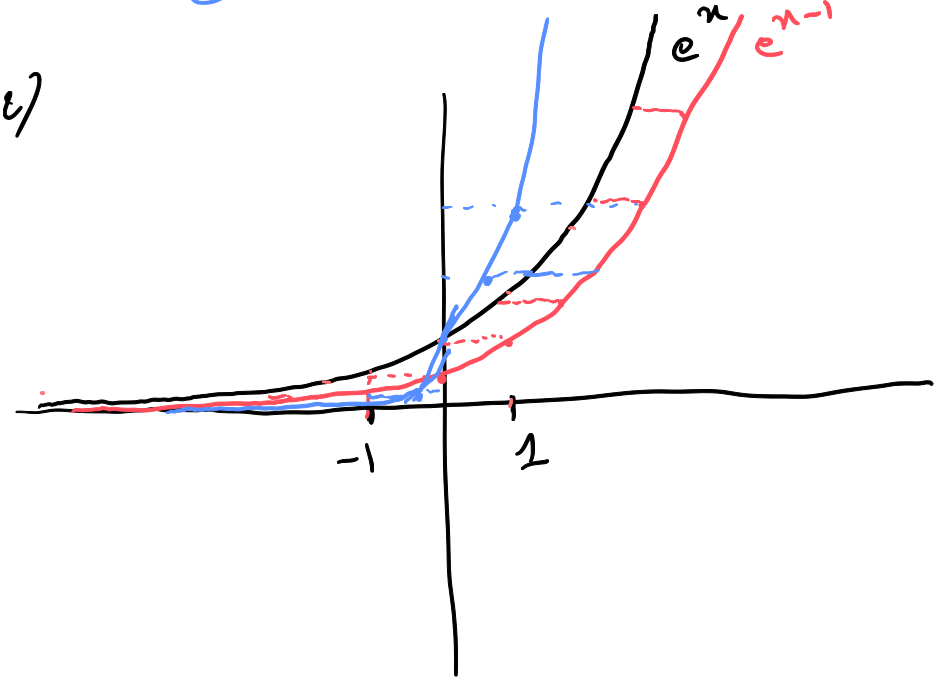
$$f(x) = (1+x)^x = e^{x \ln(1+x)}$$ $\qquad$ $Df =]-1 ; +\infty[$.

Exercice 4 :

1) $f = g \circ h = g(3x) = e^{3x-1}$ avec $g(x) = e^{x-1}$

Translation à droite de 1 : $e^x \to e^{x-1}$; dilatation horizontale $\frac{1}{3}$: $e^{x-1} \to e^{3x-1}$

2)



Exercice 5.

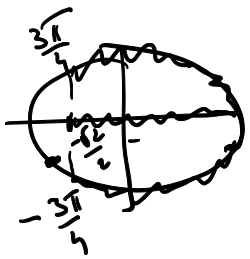
1) $\sin 2x = \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right)$

$$\begin{cases} 2x = \frac{\pi}{3} \ [2\pi] \\ \text{ou} \\ 2x = \pi - \frac{\pi}{3} \ [2\pi] \end{cases} \qquad \begin{cases} x = \frac{\pi}{6} \ [\pi] \\ \text{ou} \\ x = \frac{\pi}{3} \ [\pi] \end{cases}$$

Dans $[0; 2\pi]$: $\frac{\pi}{6}$; $\frac{7\pi}{6}$; $\frac{\pi}{3}$; $\frac{4\pi}{3}$

2) $\cos x \geq -\frac{\sqrt{2}}{2}$

$\Leftrightarrow \quad -3\frac{\pi}{4} \leq x \leq \frac{3\pi}{4} \ [2\pi]$



et $x \in \left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$ dans $[-\pi, \pi[$

3) $\tan(3x) + \cos(4x)$

$\tan 3x \longrightarrow \frac{\pi}{3}$ - periodique

$\cos(4x) \longrightarrow \frac{2\pi}{4} = \frac{\pi}{2}$ - periodique

} donc une période commune est $k\frac{\pi}{3} = k'\frac{\pi}{2}$

$\Rightarrow \frac{k}{k'} = \frac{3}{2} \Rightarrow k = 3$ et $k' = 2$

$\Rightarrow \tan 3x + \cos 4x$ est $3 \times \frac{\pi}{3} = \pi$ periodique.

Exercice 6 :

$$f(x) = \operatorname{ch}\left(\frac{x+2}{x-1}\right)$$

1) $x - 1 \neq 0$ $\quad D_f = \mathbb{R} \setminus \{1\}$ car ch est définie sur $\mathbb{R}$.

2) $\lim\limits_{\pm\infty} f(x) = \operatorname{ch}(1)$

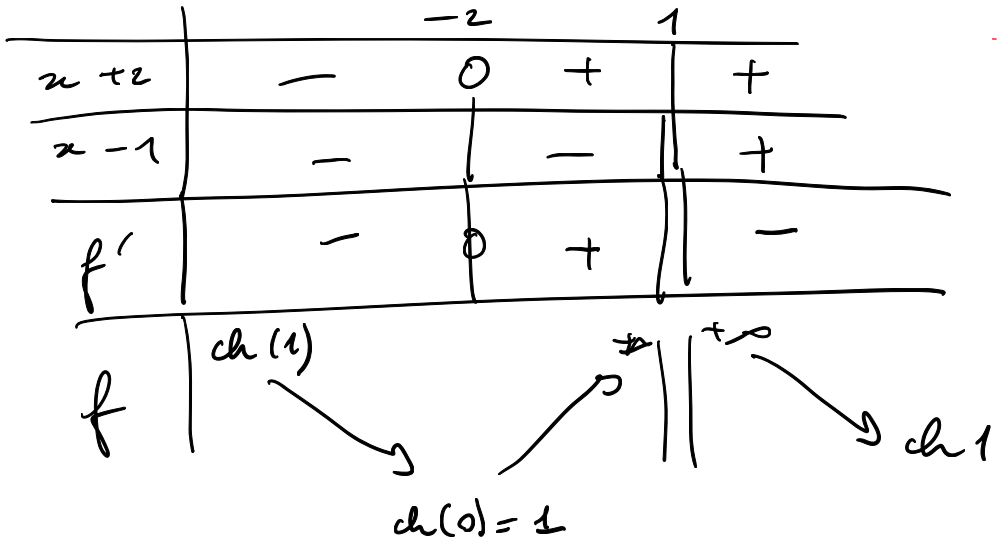
$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{X \to -\infty} \operatorname{ch} X = +\infty$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{X \to +\infty} \operatorname{ch} X = +\infty$

3) $f'(x) = \frac{(x-1) - (x+2)}{(x-1)^2} \operatorname{sh}\left(\frac{x+2}{x-1}\right)$

$= \underbrace{\frac{-3}{(x-1)^2}}_{\leq 0} \operatorname{sh}\left(\frac{x+2}{x-1}\right)$

du signe de $\frac{x+2}{x-1}$



Exercice 7 :

1] a) $2y' + 5y = x + \cos(2x)$

y_H : $y' + \frac{5}{2}y = 0$ $\quad y_H = k e^{-\frac{5}{2}x}$

y_p :

par x : $5 \mid y_{p_1} = ax + b$

$2 \mid y'_{p_1} = a$

$2y' + 5y \mid 5ax + 5b + 2a = x$

$$\begin{cases} 5a = 1 \\ 5b + 2a = 0 \end{cases} \qquad a = \frac{1}{5} \qquad b = -\frac{2}{25}$$

$$y_{p_1} = \frac{1}{5}x - \frac{2}{25}$$

Solution complète : $y = y_H + y_{p_1}$

$$= k e^{-\frac{5}{2}x} + \frac{1}{5}x - \frac{2}{25}$$

(b) $f(0)=1$:

$$y(0) = K - \frac{2}{25} = 1 \quad \Rightarrow K = 1 + \frac{2}{25} = \frac{27}{25}$$

$$\Rightarrow f(x) = \frac{27}{25} e^{-\frac{5}{2}x} + \frac{1}{5}x - \frac{2}{25}$$

2) $2y'' + 3y' + 5y = 0$

Ec: $2r^2 + 3r + 5 = 0$

$$\Delta = 9 - 40 = -31$$

$$\alpha = \frac{-3}{4} \qquad \beta = \frac{\sqrt{31}}{4}$$

$$y = \left(A \cos\frac{\sqrt{31}}{4}x + B \sin\frac{\sqrt{31}}{4}x\right) e^{-\frac{3}{4}x}$$

3) $\ddot{x} - 2\dot{x} - x = 2t^2 + \sin(3t)$

x_H: Ec: $r^2 - 2r - 1 = 0 \qquad \Delta = 4 + 4 = 8.$

$$\alpha = \frac{2+\sqrt{8}}{2} \qquad \beta = \frac{2-\sqrt{8}}{2}$$

$$x_H = Ae^{\alpha t} + Be^{\beta t}$$

x_p: para $2t^2$

$$\begin{array}{c|l} & x_{p_1} = at^2 + bt + c \\ -2 & x'_{p_1} = 2at + b \\ 1 & x''_{p_1} = 2a \end{array}$$

$$= -at^2 + (-b - 4a)t + 2a - 2b - c$$

$$\begin{cases} -a = 2 \\ -b - 4a = 1 \\ 2a - 2b - c = 0 \end{cases} \quad \begin{cases} a = -2 \\ b = 8 \\ c = -20 \end{cases} \qquad x_{p_2} = -2t^2 + 8t - 20 .$$

para $\sin(3t)$: -1

(*) com $\omega = 3$ da $i\omega$ pois é sol. de $P(EC)$

$$x_{p_2}^{(*)} = a\cos 3t + b \sin 3t$$

$$-2 \quad x'_{p_2} = 3b\cos 3t - 3a \sin 3t$$

$$1 \quad x''_{p_2} = -9a\cos 3t - 9b \sin 3t$$

$$= (-6b - 10a)\cos(3t) + (-10b + 6a)\sin 3t$$

$$\begin{matrix} L_1 \\ L_2 \end{cases} \begin{matrix} -10b + 6a = 1 \\ -6b - 10a = 0 \end{matrix}$$

$$6L_1 - 10L_2 : 136a = 6$$

$$a = \frac{6}{136}$$

$$10L_1 + 6L_2 : -136b = 10$$

$$b = \frac{-10}{136}$$

$$x_{p_2} = \frac{6}{136}\cos(3x) - \frac{10}{136}\sin(3t)$$

$$x = x_H + x_{p_1} + x_{p_2}$$

Exercício 6: $V(h) = \pi R^2 h$.

$\Delta V \simeq dV$ $\qquad dV = \pi R^2 dh = 4\pi \times 0{,}01.$

$dh = 0{,}01\ m$