Exo 1 · $z = \frac{a-b}{1+ab}$

$$\overline{\left(\frac{a-b}{1+ab}\right)} = \frac{\bar{a}-\bar{b}}{1+\bar{a}\bar{b}} = \frac{\frac{1}{a}-\frac{1}{b}}{1+\frac{1}{ab}} = \frac{b-a}{ab+1}$$

$a\bar{a} = b\bar{b} = 1$

$$= -\frac{a-b}{1+ab}$$

on aurait multiplié.

donc on a $z = -\bar{z}$ $\quad z \in i\mathbb{R}$

Exo 2 :

$$\left(\frac{z+1}{z-1}\right)^2 = -1 = i^2$$

$\Leftrightarrow \frac{z+1}{z-1} = i$ ou $\frac{z+1}{z-1} = -i$

$\Leftrightarrow z+1 = iz-i$ ou $z+1 = -iz+i$

$\Leftrightarrow z(1-i) = -1-i$ ou $z(1+i) = -1+i$

$\Leftrightarrow z = \frac{-1-i}{1-i}$ ou $z = \frac{-1+i}{1+i}$

$\Leftrightarrow z = \frac{(-1-i)(1+i)}{2}$ ou $z = \frac{(-1+i)(1-i)}{2}$

$\Leftrightarrow z = \frac{-1-i-i+1}{2}$ ou $z = \frac{-1+i+i+1}{2}$

$\Leftrightarrow z = -i$ ou $z = i$

$\Leftrightarrow z = e^{3i\pi/2}$ ou $z = e^{i\pi/2}$

Exo 3 :

$$iz^2 - 2iz + i - 2 = 0 \quad (E)$$

$\Delta = (-2i)^2 - 4 \times i(i-2)$

$= -4 + 4 + 8i$

Déterminons les racines carrées δ de Δ /

$\delta = \rho e^{i\theta}$ et $\delta^2 = \Delta$ (3)

(3) $\Leftrightarrow \delta = \rho e^{i\theta}$ et $\rho^2 e^{2i\theta} = 8e^{i\pi/2}$

(3) $\Leftrightarrow$ ——— $\begin{cases} \rho^2 = 8 \\ 2\theta = \frac{\pi}{2} + 2k\pi \quad (k=0, k=1) \end{cases}$

(3) $\Leftrightarrow$ ——— $\begin{cases} \rho = 2\sqrt{2} \text{ car } \rho > 0 \\ \theta = \pi/4 + k\pi \end{cases}$

(3) $\Leftrightarrow \delta = 2\sqrt{2}\, e^{i\pi/4}$ ou $\delta = -2\sqrt{2}\, e^{i\pi/4}$

En choisissant $\delta = 2\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= 2 + 2i$

$(E) \Leftrightarrow z = \dfrac{2i - (2+2i)}{2i}$ ou $z = \dfrac{2i + 2 + 2i}{2i}$

$(E) \Leftrightarrow z = -\dfrac{1}{i} = i$ ou $z = \dfrac{2+4i}{2i} = \dfrac{1+2i}{i}$

$z = 2 - i$

$$\boxed{S = \{ i \ ; \ 2-i \}}$$

Exo 4 :

$$\frac{1}{z} = \frac{1}{jR\omega} + \frac{1}{LC\omega} = \frac{LC\omega + jR\omega}{jRLC\omega^2}$$

$$\Leftrightarrow z = \frac{jRLC\omega^2}{LC\omega + jR\omega} = \frac{jRLC\omega}{LC + jR}$$

$$|z| = \frac{|jRLC\omega|}{|LC + jR|} = \boxed{\frac{RLC\omega}{\sqrt{(LC)^2 + R^2}}}$$

$$\arg(z) = \arg(jRLC\omega) - \arg(LC + jR)$$

$$jRLC\omega = RLC\omega \, e^{j \cdot \pi/2}$$

$$LC + jR = \sqrt{(LC)^2 + R^2} \left(\frac{LC}{\sqrt{(LC)^2 + R^2}} + j \frac{R}{\sqrt{(LC)^2 + R^2}} \right)$$

$$= \sqrt{(LC)^2 + R^2} \; e^{j\varphi} \quad / \quad \begin{cases} \cos\varphi = LC / \sqrt{(LC)^2 + R^2} \\ \sin\varphi = R / \sqrt{(LC)^2 + R^2} \end{cases}$$

ainsi $\boxed{\arg(z) = \frac{\pi}{2} - \varphi \ [2\pi]}$

Autre méthode : $z = \dfrac{jRLC\omega(LC - jR)}{(LC)^2 + R^2} = \dfrac{RLC\omega}{(LC)^2 + R^2}(R + LCj)$

$\arg(z) = \arctan\left(\frac{LC}{R}\right)[\pi] = \arctan\left(\frac{LC}{R}\right)[2\pi]$

car $\cos > 0$ et $\sin > 0$

Exo 5 :

$$A = \cos x \, (\sin x)^3$$

$$= \left(\frac{e^{ix} + e^{-ix}}{2}\right) \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^3$$

$$= \frac{1}{2} \times \frac{1}{2^3} \times \frac{1}{i^3} \left(e^{ix} + e^{-ix}\right)\left(e^{3ix} - 3e^{2ix}e^{-ix} + 3e^{ix}e^{-2ix} - e^{-3ix}\right)$$

$$= \frac{1}{16} \times \frac{-1}{i} \left[e^{4ix} - 3e^{2ix} + 3 - e^{-2ix} + e^{2ix} - 3 + 3e^{-2ix} - e^{-4ix}\right]$$

$$= \frac{-1}{16i} \left[2i \sin(4x) + 2i \sin(2x) - 3 \times 2i \sin(2x)\right]$$

$$= \frac{-1}{16i} \left[2i \sin(4x) - 4i \sin(2x)\right]$$

$$\boxed{A = -\frac{1}{8} \sin(4x) + \frac{1}{4} \sin(2x)}$$

Exo 6 :

1) $\begin{cases} X = 2x - 3y \\ Y = x + y \end{cases} \Leftrightarrow \begin{cases} X - 2Y = 2x - 3y - 2x - 2y \\ X + 3Y = 2x - 3y + 3x + 3y \end{cases} \Leftrightarrow \begin{cases} x = \frac{X + 3Y}{5} \\ y = \frac{-X + 2Y}{5} \end{cases}$

2) $\frac{\partial g}{\partial y} = \frac{-X}{25} + \frac{7Y}{25} \Rightarrow g(X,Y) = \frac{-XY}{25} + \frac{7}{25} \times \frac{Y^2}{2} + A(X)$ où A fonct° C¹ de x

3) $3\frac{\partial f}{\partial x} + 2\frac{\partial f}{\partial y} = x + 2y \quad (E)$

$(x,y) \longrightarrow (X = 2x - 3y, Y = x + y) \longrightarrow g(X,Y) = f(x,y)$

Par les règles de composition :

$$\frac{\partial f}{\partial x} = \frac{\partial X}{\partial x}\frac{\partial g}{\partial X} + \frac{\partial Y}{\partial x}\frac{\partial g}{\partial Y} = 2\frac{\partial g}{\partial X} + \frac{\partial g}{\partial Y}$$

$$\frac{\partial f}{\partial y} = \frac{\partial X}{\partial y}\frac{\partial g}{\partial X} + \frac{\partial Y}{\partial y}\frac{\partial g}{\partial Y} = -3\frac{\partial g}{\partial X} + \frac{\partial g}{\partial Y}$$

De ① par 1) $\quad x+2y = \frac{X+3Y}{5} + 2\times\frac{-X+2Y}{5}$

$$= \frac{(X-2X)+3Y+4Y}{5} = \frac{-X+7Y}{5}$$

$$(E) \Leftrightarrow 6\frac{\partial g}{\partial X} + 3\frac{\partial g}{\partial Y} - 6\frac{\partial g}{\partial X} + 2\frac{\partial g}{\partial Y} = \frac{-X+7Y}{5}$$

$$(E) \Leftrightarrow \quad 5\frac{\partial g}{\partial Y} = -\frac{X}{5} + \frac{7}{5}Y$$

$$(E) \Leftrightarrow \quad \frac{\partial g}{\partial Y} = -\frac{X}{25} + \frac{7}{25}Y$$

$$(E) \Leftrightarrow \text{(par Q2)} \quad g(X,Y) = -\frac{XY}{25} + \frac{7}{25}\frac{Y^2}{2} + A(X) \text{ où } A \text{ fonct}^{\text{n}} C^1$$

$$(E) \Leftrightarrow \quad f(x,y) = -\frac{(2x-3y)(x+y)}{25} + \frac{7}{50}\times(x+y)^2 + A(2x-3y) \text{ où } A \text{ fonct}^{\text{n}} C^1$$

Exo 7 :

1) $f(x,y) = 3yx^2$

$$\frac{\partial f}{\partial x}(x,y) = 6yx \qquad \frac{\partial f}{\partial y}(x,y) = 3x^2$$

$$df_{(4,3)}(h,k) = \frac{\partial f}{\partial x}(4,3)\times h + \frac{\partial f}{\partial y}(4,3)\times k$$

$$\boxed{df_{(4,3)}(h,k) = 72h + 48k}$$

2) Par déf f est différentiable en $(4,3)$ s'il existe des réels m et p et un disque $\mathcal{D}$ centré en $(4,3)$ / $\forall (h,k) / (4+h,3+k)\in\mathcal{D}$:

$$f(4+h,3+k) = f(4,3) + mh + p\cdot k + \|(h,k)\|\times\varphi(4+h,3+k)$$

avec $\lim_{(h,k)\to(0,0)} \varphi(4+h,3+k) = 0$

3) Ici f est C^1 comme fonction polynomiale donc f est différentiable.

Autre méthode

$f(4+h,3+k) - f(4,3) - 72h - 48k = 3(3+k)(4+h)^2 - 3\times16\times3 - 72h - 48k$

$= (9+3k)(16+8h+h^2) - 3\times16\times3 - 72h - 48k = 9h^2 + 24hk + 3kh^2$

4) L'équation du plan tangent à la surface est

$$z = f(4,3) + (x-4)\frac{\partial f}{\partial x}(4,3) + (y-3)\frac{\partial f}{\partial y}(4,3)$$

$\left|\sqrt{h^2+k^2}\right| \left|\frac{9h^2+24hk+3kh^2}{\sqrt{h^2+k^2}}\right|$

→ Par Pair $\mathcal{R}(\mathcal{R}(A))$ → 0 → 0

à savoir $z = 108 + 72(x-4) + 48(y-3)$

de vecteur normal $\vec{n}\begin{pmatrix}72\\48\\-1\end{pmatrix}$

or (P): $2x-5y+z=0$ a pour vecteur normal $\begin{pmatrix}2\\-5\\1\end{pmatrix}$

Ces 2 vecteurs normaux n'étant pas colinéaires, les plans ne sont pas $/\!/$.

Exo 8 :

1) $f(r,\theta) = g(x,y) = g(r\cos\theta, r\sin\theta)$

$(r,\theta) \longrightarrow (\overset{x=}{r\cos\theta}, \overset{y=}{r\sin\theta}) \longrightarrow g(x,y) = f(r,\theta)$

$$\frac{\partial f}{\partial \theta}(r,\theta) = \frac{\partial x}{\partial \theta}\frac{\partial g}{\partial x} + \frac{\partial y}{\partial \theta}\frac{\partial g}{\partial y}$$

$$\frac{\partial f}{\partial \theta} = (-r\sin\theta)\frac{\partial g}{\partial x} + (r\cos\theta)\frac{\partial g}{\partial y}$$

ainsi $$\boxed{\frac{\partial f}{\partial \theta} = -y\frac{\partial g}{\partial x} + x\frac{\partial g}{\partial y}}$$

2) $$\frac{\partial^2 f}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right)$$

$$= \frac{\partial}{\partial \theta}\left[(-r\sin\theta)\frac{\partial g}{\partial x} + (r\cos\theta)\frac{\partial g}{\partial y}\right]$$

$$= (-r\cos\theta)\frac{\partial g}{\partial x} + (-r\sin\theta)\frac{\partial}{\partial \theta}\left(\frac{\partial g}{\partial x}\right) + (-r\sin\theta)\frac{\partial g}{\partial y} + r\cos\theta\frac{\partial}{\partial \theta}\left(\frac{\partial g}{\partial y}\right)$$

$$= (-r\cos\theta)\frac{\partial g}{\partial x} + (-r\sin\theta)\left[\frac{\partial x}{\partial \theta}\frac{\partial^2 g}{\partial x^2} + \frac{\partial y}{\partial \theta}\frac{\partial^2 g}{\partial y \partial x}\right]$$

$$+ (-r\sin\theta)\frac{\partial g}{\partial y} + (r\cos\theta)\left[\frac{\partial x}{\partial \theta}\frac{\partial^2 g}{\partial x \partial y} + \frac{\partial y}{\partial \theta}\frac{\partial^2 g}{\partial y^2}\right]$$

$$= -x\frac{\partial g}{\partial x} - y\frac{\partial g}{\partial y} + (-r\sin\theta)\left[-r\sin\theta\frac{\partial^2 g}{\partial x^2} + r\cos\theta\frac{\partial^2 g}{\partial y \partial x}\right]$$

$$+ (r\cos\theta)\left[-r\sin\theta\frac{\partial^2 g}{\partial x \partial y} + r\cos\theta\frac{\partial^2 g}{\partial y^2}\right]$$

$$\boxed{\frac{\partial^2 f}{\partial \theta^2} = -x\frac{\partial g}{\partial x} - y\frac{\partial g}{\partial y} + y^2\frac{\partial^2 g}{\partial x^2} - 2xy\frac{\partial^2 g}{\partial x \partial y} + x^2\frac{\partial^2 g}{\partial y^2}}$$

par Thm Schwarz