DS 1 M 1.1 Correction.

Exercice 1 :

1) $\forall \varepsilon > 0 \quad \exists M > 0 \;/\; \forall x \in I \quad x \geq M \Rightarrow |f(x) - l| \leq \varepsilon$

2) Soit $\varepsilon = \frac{l}{2}$ $\exists A > 0 \;/\; \forall x \in I \quad x \geq M \quad |f(x) - l| \leq \frac{l}{2}$

ainsi $-\frac{l}{2} \leq f(x) - l \leq \frac{l}{2}$

$$0 < \frac{l}{2} \leq f(x) .$$

Exercice 2 :

1) $\lim\limits_{x \to 2} x^2 - 4 = 0 = \lim\limits_{x \to 2}$ $x^2 + x - 6$ F.I.

$x^2 - 4 = (x+2)(x-2)$ $\quad x^2 + x - 6 = (x-2)(x+3)$

$$① = \lim_{x \to 2} \frac{x+2}{x+3} = \frac{4}{5}$$

2) $\lim\limits_{x \to 0} \dfrac{\ln(1+2x)}{3x}$ $\quad u = 2x$

$$② = \lim_{u \to 0} \frac{\ln(1+u)}{\frac{3}{2}u} = \frac{2}{3} \lim_{u \to 0} \frac{\ln(1+u)}{u} = \frac{2}{3}$$

3) $\lim\limits_{R \to +\infty} \dfrac{RCw}{1+RCw} = \lim\limits_{R \to +\infty} \dfrac{RCw}{RCw} = 1$

4) $\sqrt{2x+5} - \sqrt{x^2+3} = \underbrace{\sqrt{x}}_{+\infty} \left(\underbrace{\sqrt{2 + \frac{5}{x}}}_{\to \sqrt{2}} - \underbrace{\sqrt{x + \frac{3}{x}}}_{\to +\infty} \right)$

$\lim\limits_{x \to +\infty} f = -\infty$

5) Si $a > 0$ $\lim\limits_{x \to -\infty} e^{ax} = 0$ et $\lim f(x) = -2$

Si $a = 0$ $f(x) = \frac{3}{2}$ Si $a < 0$ $f(x) = \dfrac{e^{ax}\left(1 + \frac{2}{e^{ax}}\right)}{3e^{ax}\left(3 - \frac{1}{e^{ax}}\right)}$ et $\lim\limits_{x \to -\infty} f(x) = \frac{1}{3}$

Exercice 3 :

1] $|-5x+1| < 3 \iff -3 < -5x+1 < 3$

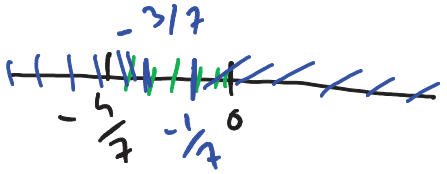
$\iff -4 < -5x < 2$

$\iff \frac{4}{5} > x > -\frac{2}{5}$ $\qquad x \in]-\frac{2}{5}; \frac{4}{5}[$.

2] $1 \leq |7x+2| \leq 2 \iff |7x+2| \leq 2$ (A) et $1 \leq |7x+2|$ (B)

(A) $-2 \leq 7x+2 \leq 2$ et (B) $7x+2 \geq 1$ ou $7x+2 \leq -1$

$-\frac{4}{7} \leq x \leq 0$ $\qquad x \geq -\frac{1}{7}$ ou $x \leq -\frac{3}{7}$

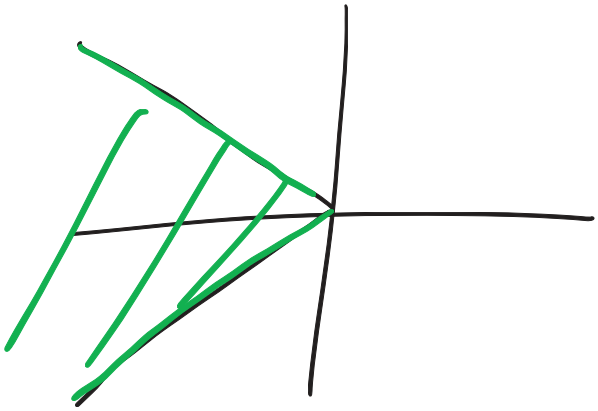


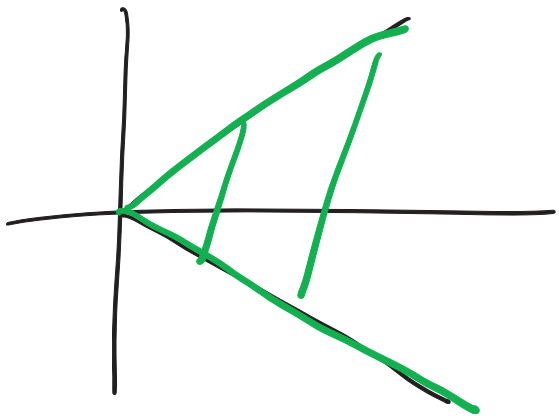
A et B : $x \in [-\frac{4}{7}; -\frac{3}{7}] \cup [-\frac{1}{7}; 0]$

3] $|\frac{y}{x}| \leq 1 \iff -1 \leq \frac{y}{x} \leq 1$ et $x \neq 0$

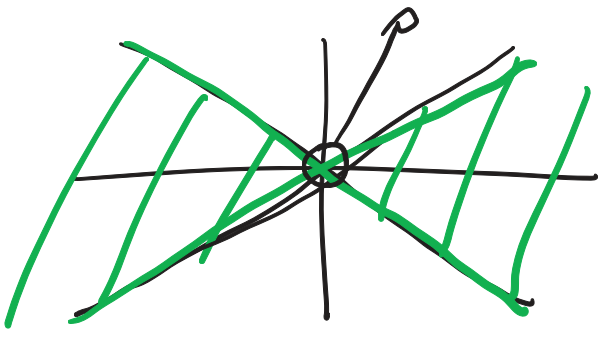
si $x > 0 \iff -x \leq y \leq x$

si $x < 0 \iff -x \geq y \geq x$





Bilan :



Exercice 4 :

$f(x) = x^2 e^{-1/x}$ $\quad x \in]0;+\infty[$

1) $\lim\limits_{x \to 0} f(x)$ $\quad \lim\limits_{x \to 0^+} e^{-1/x} = \lim\limits_{X \to -\infty} e^X = 0.$ 5

$\lim\limits_{x \to 0^+} f(x) = 0$. on peut donc prolonger f par continuité en 0. en posant $f(0) = 0$.

2) $\tau_0(h) = \dfrac{f(h) - f(0)}{h} = \dfrac{h^2 e^{-1/h}}{h} = h e^{-1/h}$

$\lim\limits_{h \to 0} \tau_0(h) = 0$ donc f est dérivable en 0 et $f'(0) = 0$

3) $f'(x) = 2x e^{-1/x} + x^2 \times \left(+\frac{1}{x^2}\right) e^{-1/x}$

$= e^{-1/x}(2x+1)$

$\lim\limits_{x \to 0} f'(x) = 0 = f'(0)$ donc f' est continue en 0, donc f est de classe C^1 en 0 (0,25) ,25

Exercice 5 :

$f(x) = \sin(|x|)$

1) sur $\mathbb{R}_+^*$ $f(x) = \sin x$ f est dérivable.

sur $\mathbb{R}_-^*$ $f(x) = \sin(-x)$ ———

en 0 : $f(0) = \sin 0 = 0$

$\tau_0(h) = \dfrac{f(h) - f(0)}{h} = \dfrac{\sin(|h|)}{h}$

$$\lim_{0^+} \tau_0(h) = \lim_{0^+} \frac{\sin h}{h} = 1 \neq$$

$$\lim_{0^-} \tau_0(h) = \lim_{0^-} \frac{\sin(-h)}{h} = \lim_{v \to 0^+} -\frac{\sin v}{v} = -1$$

$v = -h$

f n'est pas dérivable en 0.

(C) C_f admet 2 demi tangente de pentes 1 et -1. ✳

Exercice 6 :

$$f(x) = \sqrt{\frac{3\ln(2x+1)-5}{4\ln(2x+1)+2}} \qquad f'(x) = \frac{1}{2}\left(\frac{3\ln(2x+1)-5}{4\ln(2x+1)+2}\right)' \underbrace{\left(\quad\right)^{-1/2}}_{④}$$

$$④ = (\ln(2x+1))' \times \left(\frac{3X-5}{4X+2}\right)'_{\ln(2x+1)} = \frac{2}{2x+1} \times \frac{26}{(4(\ln(2x+1)+2)^2}$$

$$f'(x) = \frac{26}{(2x+1)} \times \frac{1}{(4\ln(2x+1)+2)^2} \times \sqrt{\frac{4\ln(2x+1)+2}{3\ln(2x+1)-5}}$$

Exercice 7 :

$$r = \frac{\cos y}{\sin x + 2}$$

$$dr = \frac{d(\cos y).(\sin x+2) - \cos y \cdot d(\sin x+2)}{(\sin x+2)^2}$$

$$= \frac{(-\sin y)(\sin x+2)\,dy - \cos y \cdot \cos x\, dx}{(\sin x+2)^2}$$

et $dy = y'dt \qquad dx = x'dt.$

$$= \frac{\left(-y'\sin y(\sin x+2) - x'\cos y \cos x\right)dt}{(\sin x+2)^2}$$

Exercice 8 : $V = n\frac{RT}{P}$

1) $\Delta V = V_f - V_i = \frac{nRT}{200100} - \frac{nRT}{200000}$

2) $\Delta V \approx dV = -\frac{nRT}{P^2} dP$

avec $P = 200000$ $\Delta P = 100$.